II KONFERENCJA

MATEMATYCZNO – INFORMATYCZNA

„CONGRESSIO MATHEMATICA”

Mierki, 20 – 23.09.2016

Katedra Analizy Zespolonej
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MATEMATYCZNO – INFORMATYCZNA

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ORGANIZATORZY

Katedra Analizy Zespolonej
Wydział Matematyki i Informatyki
Uniwersytet Warmińsko-Mazurski w Olsztynie

Wydział Matematyczno – Przyrodniczy
Uniwersytet Rzeszowski

Honorowy Patronat
nad tegoroczną konferencją objęli:

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Uniwersytetu Warmińsko – Mazurskiego w Olsztynie
Prof. dr hab. Ryszard GÓRECKI

JM Rektor
Uniwersytetu Rzeszowskiego
Prof. dr hab. Sylwester CZOPEK

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RAMOWY PROGRAM KONFERENCJI

Wtorek, 20 września 2016

12:00 – 17:00 Rejestracja uczestników konferencji
(Hol główny Hotelu Kormoran)

13:00 – 16:00 Obiad

19:00 – 00:00 Kolacja powitalna

Środa, 21 września 2016

07:30 – 09:00 Śniadanie

09:15 – 10:05 Wykład plenarny
10:15 – 11:05 Wykład plenarny

11:25 – 12:50 Sekcja referatów

13:00 – 14:00 Obiad

14:30 – 15:20 Wykład plenarny
15:30 – 16:20 Wykład plenarny

16:40 – 18:05 Sekcja referatów

19:00 – ??:?? Bankiet

Czwartek, 22 września 2016

07:30 – 09:00 Śniadanie

09:15 – 10:05 Wykład plenarny
10:15 – 11:05 Wykład plenarny

11:25 – 12:50 Sekcja referatów

13:00 – 14:00 Obiad

14:30 – 15:20 Wykład plenarny
15:30 – 16:20 Wykład plenarny

16:40 – 18:35 Sekcja referatów

19:00 – ??:?? Biesiada grillowa

Piątek, 23 września 2016

07:30 – 09:00 Śniadanie

09:15 – 10:05 Wykład plenarny
10:15 – 11:05 Wykład plenarny

11:15 – 12:15 Sesja posterowa
(Hol główny Hotelu Kormoran)

13:00 – 14:00 Obiad

Marek ALEKSIEJCZYK

University of Warmia and Mazury in Olsztyn (Olsztyn)

Magic matrices - results and questions

We consider so called magic matrices, i.e., square matrices of natural entries with equal sums in each row, column, main diagonal and antidiagonal. We focus our attention on

- squares, higher powers (we prove that they are getting "relatively symmetric") ,

- numerical ranges of magic matrices (a numerical range of a square matrix A is a subset of the complex plane $W(A) = \{\langle Ax, x\rangle : \langle x, x\rangle = 1\}$),

- powers (we present formulaes for numerical ranges of all 3×3 magic matrices of order $4n$ generated by MATLAB).

Some open problems are also proposed.

Agnieszka BOJARSKA-SOKOŁOWSKA

University of Warmia and Mazury in Olsztyn (Olsztyn)

Geometryczne eksperymentowanie przez dzieci, młodzież i dorosłych

W swoim wystąpieniu przedstawię warsztaty matematyczne polegające na eksperymentowaniu. Eksperymenty te dotyczyły składania z klocków obiektów matematycznych takich jak: parkietaże, wielościany platońskie i archimedesowe. Uczestnicy zajęć odkrywali własności takie jak: układanie tych obiektów na podstawie kodu na wierzchołek, wzór Eulera, warunki dostateczne do stworzenia parkietaży archimedesowych itp. Warsztaty te prowadzone dla dzieci (II-III klasy szkoły podstawowej), młodzieży (II-III klasa gimnazjum), studentów matematyki oraz nauczycielek edukacji wczesnoszkolnej i nauczycieli matematyki. Każde z tych zajęć zostało specjalnie dostosowane do konkretnych odbiorców.

Geometric experimentation by children, youth and adults

In my speech I shall present a workshop involving mathematical experimentation. These experiments involved the construction of blocks of mathematical objects such as tessellations, polyhedra Platonic and Archimedean. Participants discovered properties such as the composition of these objects on the basis of code on top, Euler formula, sufficient conditions to construct Archimedean tessellations, etc. These workshops were conducted for children (II-III grade of primary school), youth (II-III class of gymnasium), students of mathematics, teachers of early childhood education and teachers of mathematics. Each of the classes has been specially adapted to specific audience.

Oliwia CHOJNACKA, Adam LECKO

University of Warmia and Mazury in Olsztyn (Olsztyn)

Differential subordination of the harmonic mean to a linear function

Given $\beta \in [0,1]$ and a, $b \in \mathbf{C}$ such that $b+\beta(b-a) \neq 0$, the harmonic mean of a and b is defined as $ab/(b+\beta(a-b))$. For $\beta \in (0,1]$ and $M > 0$ we discuss the differential subordination

$$\frac{p(z)(p(z)+zp'(z)\Phi(p(z)))}{p(z)+(1-\beta)zp'(z)\Phi(p(z))} \prec 1+Mz, \quad z \in \mathbf{D},$$

in case when $\Phi \equiv 1$ and when $\Phi(w) := 1/w,\ w \in \mathbf{C} \setminus \{0\}$. The range of M for which the above condition yields

$$p(z) \prec 1+Mz, \quad z \in \mathbf{D},$$

is computed [3] (see also [1] and [2]).

References

[1] N. E. Cho, A. Lecko, *Differential subordination of harmonic mean*, Comput. Methods Funct. Theory **15** (2015), no. 3, 427-437.
[2] O. Chojnacka, A. Lecko, *On differential subordination of harmonic mean*, Bull. Soc. Sci. Lett. Łódź. Série: Recherches sur les Déformations **64** (2014), no. 2, 29-40.
[3] O. Chojnacka, A. Lecko, *On the differential subordination of harmonic mean to a linear function*, Bull. Soc. Sci. Lett. Łódź. Série: Recherches sur les Déformations **66** (2016), no. 2.

Jacek CHUDZIAK

University of Rzeszów (Rzeszów)

Quasideviation means in insurance

Quasideviation means, introduced in [6], are generalizations of the deviation means, investigated in [2]-[4]. A related concept is that of implicit means, studied in [5]. A series of properties of quasidevaition means have been proved in [7]. We prove that an important notion of insurance mathematics, namely the zero utility principle introduced in [1], is a particular case of the quasideviation mean. Furthermore, we obtain the results concerning comparison, equality, positive homogeneity, subadditivity and convexity of the zero utility principles.

References

[1] H. Bühlmann, *Mathematical Models in Risk Theory*, Springer-Verlag, New York 1970.
[2] Z. Daróczy, *Über eine Klasse von Mittelwerten*, Publ. Math. Debrecen **19** (1972), 211-217.
[3] Z. Daróczy, *A general inequality for means*, Aequationes Math. **7** (1972), 16-21.
[4] Z. Daróczy, Zs. Páles, *On comparison of mean values*, Publ. Math. Debrecen **29** (1982), 107-115.
[5] L. Losonczi, *General inequalities for nonsymmetric means*, Aequationes Math. **9** (1973), 221-235.
[6] Zs. Páles, *Characterization of quasideviation means*, Acta. Math. Sci. Hungar. **40** (1982), 243-260.
[7] Zs. Páles, *General inequalities for quasideviation means*, Aequationes Math. **36** (1988), 32-56.

Małgorzata CHUDZIAK, Barbara SOBEK

University of Rzeszów (Rzeszów)

Pexider type equation on a restricted domain

Inspired by the papers by Aczél [3]-[4] and by Chudziak and Tabor [5], we consider the problem of existence and uniqueness of extensions for the Pexider type equation

$$k(x+y) = l(x) + m(x)n(y) \quad \text{for} \quad (x,y) \in D, \tag{1}$$

where D is a nonempty open subset of a normed space. In particular, we show that the connectedness of D, assumed in the mentioned above papers, can be weakened. Equation (1), as well as some of its particular cases, play a crucial role in solving various problems in utility theory and decision analysis. For more details concerning such applications we refer to [2] and to a survey paper [1]. Furthermore, the particular case $n = k$ of (1) was used to prove that the power means and the geometric mean are the only homogeneous quasiarithmetic means (cf. [6]).

References

[1] A. E. Abbas, J. Aczél, *The role of some functional equations in decision analysis*, Decision Analysis **7** (2010), 215-228.
[2] A. E. Abbas, J. Aczél, J. Chudziak, *Invariance of multiattribute utility functions under shift transformations*, Result. Math. **54** (2009), 1-13.
[3] J. Aczél, *Utility of extension of functional equations – when possible*, J. Math. Psych. **49** (2005), 445-449.
[4] J. Aczél, *Extension of a generalized Pexider equation*, Proc. Amer. Math. Soc. **133** (2005), 3227-3233.
[5] J. Chudziak, J. Tabor, *Generalized Pexider equation on a restricted domain*, J. Math. Psych. **52** (2008), 389-392.
[6] G. H. Hardy, J. E. Littlewood, G. Polya, *Inequalities*, Cambridge University Press, Cambridge 1952.

Renata DŁUGOSZ, Piotr LICZBERSKI

Lodz University of Technology (Łódź)

Some relations between starlikeness, convexity and k-symmetry of locally biholomorphic maps in $\mathbf{C}^n$

In $\mathbf{C}^n$ let us denote by $\langle \cdot, \cdot \rangle$ the Euclidean inner product and by $\mathbf{B}^n$ the open unit ball. Many authors considered the families St, Sc of biholomorphic mappings $f : \mathbf{B}^n \to \mathbf{C}^n$, $f(0) = 0$, $Df(0) = I$, with starlike and convex domain $f(\mathbf{B}^n)$, respectively (I is the identity matrix). Suffridge [Su] proved that a locally biholomorphic normalized map $f : \mathbf{B}^n \to \mathbf{C}^n$ belongs to St iff

$$\operatorname{Re}\left\langle [Df(z)]^{-1} f(z), z \right\rangle > 0, \quad z \in \mathbf{B}^n \setminus \{0\}. \tag{1}$$

The subject of the lecture are inclusions between families St, Sc and a family $S(k)$, $k \geqslant 2$, of locally biholomorphic mappings defined similarly to (1). We use a unique partition

[LP] $f = \sum_{j=0}^{k-1} f_{j,k}$ with components $f_{j,k}$ such that

$$f_{j,k}(\varepsilon z) = \varepsilon^j f_{j,k}(z), \quad z \in \mathbf{B}^n,$$

where $\varepsilon = \exp(2\pi i//k)$. Let $S(k)$, $k \geqslant 2$, be the family of locally biholomorphic and normalized mappings $f : \mathbf{B}^n \to \mathbf{C}^n$ such that

$$\operatorname{Re}\left\langle [Df(z)]^{-1} f_{1k}(z), z \right\rangle > 0, \quad z \in \mathbf{B}^n \setminus \{0\}.$$

A motivation for the family $S(k)$ comes from the papers [HK] and [Lic]. The family $S(k)$ is a superset of a family which is connected with a problem possesed in the paper [Lic] and solved in [HK].

References

[HK] H. Hamada, G. Kohr, *k-fold symmetrical mappings and Loewner chains*, Demonstratio Math. **40** (2007), 85-94.

[Lic] P. Liczberski, *Applications of a decomposition of holomorphic mappings in* $\mathbf{C}^n$ *with respect to a cyclic group*, J. Math. Anal. Appl. **281** (2003), 276-286.

[LP] P. Liczberski, J. Połubiński, *On* (j,k)*-symmetrical functions*, Math. Bohemica **120**(1) (1995), 13-28.

[Su] T. J. Suffridge, *The principle of subordination applied to functions of several variables*, Pacific J. Math. **33** (1970), 241-248.

Jacek DZIOK

University of Rzeszów (Rzeszów)

Classes of harmonic functions defined by subordination

Harmonic functions are famous for their use in the study of minimal surfaces and also play important roles in a variety of problems in applied mathematics. Recent interest in harmonic complex functions has been triggered by geometric function theorists Clunie and Sheil-Small [1].

A complex-valued continuous function $f : D \to \mathbf{C}$ is said to be harmonic in $D \subset \mathbf{C}$ if both functions $u := \operatorname{Re} f$ and $v := \operatorname{Im} f$ are real-valued harmonic functions in D. In any simply connected domain, we can write $f = h + \overline{g}$, where h and g are analytic in D.

The object of the present talk is to define and study classes of complex-valued harmonic functions with varying coefficients related to Janowski functions.

References

[1] J. Clunie, T. Sheil-Small, *Harmonic univalent functions*, Ann. Acad. Sci. Fenn. Ser. A. I. Math. **9** (1984), 3-25.

Richard FOURNIER

Université de Montréal (Montréal, Canada)

Various proofs of Jack's lemma

I shall briefly discuss certain proofs of Jack's lemma and present, in the polynomial case, a new proof as well as an extension based on a 1982 inequality due to Stephan Ruscheweyh.

Marek GOLASIŃSKI

University of Warmia and Mazury in Olsztyn (Olsztyn)

Melvin HENRIKSEN

Clarement, U.S.A.

Residue class rings of analytic and entire functions

Let $\mathcal{A}(\mathbf{R})$ and $\mathcal{E}(\mathbf{R})$ denote respectively the ring of analytic and real entire functions in one variable. It is shown that if $\mathbf{m}$ is a maximal ideal of $\mathcal{A}(\mathbf{R})$, then $\mathcal{A}(\mathbf{R})/\mathbf{m}$ is isomorphic either to the reals or a real closed field that is an η_1-set, while if $\mathbf{m}$ is a maximal ideal of $\mathcal{E}(\mathbf{R})$, then $\mathcal{E}(\mathbf{R})/\mathbf{m}$ is isomorphic to one of the latter two fields or to the field of complex numbers. Moreover, we study the residue class rings of prime ideals of these rings and their Krull dimensions. Use is made of a classical characterization of algebraically closed fields due to E. Steinitz and techniques described in L. Gillman and M. Jerison's book on rings of continuous functions.

Mathematics Subject Classifications (2010): primary 26E05, 30D20; secondary 12D15, 13B30.
Key words and phrases: adequate ring, algebraically (real) closed field, analytic (entire) function, Bézout domain, filter, integral extension, Krull dimension, maximal (prime) ideal.

Krzysztof KAMIŃSKI

Gdynia Maritime University (Gdynia)

The Weyl – von Neumann theorem in II_∞ factor

In a separable Hilbert space $\mathcal{H}$, let $\mathcal{M}$ be a von Neumann factor of type II_∞ with semifinite normal trace τ. The q-th Schatten norm $\|\cdot\|_q$ of $A \in \mathcal{M}$ is defined as follows: $\|A\|_q = (\tau(|A|^q))^{\frac{1}{q}}$.

We will prove that for every self-adjoint operator $H \in \mathcal{M}$ and every $\varepsilon > 0$ there exists a self-adjoint perturbation operator $A \in \mathcal{M}$ such that $\|A\|_q < \varepsilon$ and $H + A$ has a pure point spectrum.

Let $|||\cdot|||$ denote an arbitrary norm, defined on a linear subspace of $\mathcal{M}$, which is invariant under automorphisms given by unitary operators from the algebra. We will prove that for every self-adjoint operator $H \in \mathcal{M}$ and every $\varepsilon > 0$ there exists a self-adjoint perturbation operator $A \in \mathcal{M}$ such that $|||A||| < \varepsilon$ and $H + A$ has a pure point spectrum. These issues are important in scattering theory and in statistical physics.

Mathematics Subject Classifications (2010): primary 47A55; secondary 47A40, 47C15, 81Q10.
Key words and phrases: von Neumann factor, perturbation of linear operator, pure point spectrum, unitarily invariant norm.

Bogumiła KOWALCZYK, Adam LECKO

University of Warmia and Mazury in Olsztyn (Olsztyn)

Coefficient functionals on some classes of analytic functions

Basing on Laguerre's rule [5] of counting zeros of real polynomials in a given interval we present sharp bounds of certain coefficient functionals on classes of analytic functions [1]-[4].

References

[1] B. Kowalczyk, A. Lecko, *Fekete-Szegö problem for close-to-convex functions with respect to the Koebe function*, Acta Math. Sci. **34**(B) (2014), no. 5, 1571-1583.
[2] B. Kowalczyk, A. Lecko, *Fekete-Szegö Problem for a Certain Subclass of Close-to-convex Functions*, Bull. Malay. Math. Sci. Soc. **38** (2015), no. 4, 1393-1410.
[3] B. Kowalczyk, A. Lecko, *Fekete-Szegö problem for close-to-convex functions with respect to a certain convex function dependent on a real paramter*, Front. Math. China **11** (2016), no. 6, 1471-1500.
[4] B. Kowalczyk, A. Lecko, *On some coefficient inequality in the subclass of close-to-convex functions*, Bull. Soc. Sci. Lett. Łódź. Série: Recherches sur les Déformations **66** (2016) (to appear).
[5] E. N. Laguerre, *Sur la théeorie des équations numériques*, J. Math. Pures et Appl. **9** (1883), 99-146 (in Oeuvres de Laguerre, vol. 1, Paris (1898), 3-47).

Joanna KOWALCZYK, Edyta TRYBUCKA

University of Rzeszów (Rzeszów)

Second Hankel determinant for certain subclass of close-to-convex functions

Let $\mathcal{K}_s(\gamma)$, $\gamma \in [0, 1)$, denote the class of analytic functions f in the open unit disc $\mathcal{U}$ with the normalization $f(0) = f'(0) - 1 = 0$ and satisfying the condition

$$\mathfrak{Re}\left[\frac{-z^2 f'(z)}{g(z)g(-z)}\right] > \gamma, \quad z \in \mathcal{U},$$

for some function g starlike of order $1/2$. For the class $\mathcal{K}_s(\gamma)$ the upper bound of the second Hankel determinant is found. Moreover the Fekete-Szegö problem is also studied.

References

[1] L. Bieberbach, *Aufstellung und Beweis des Drehungssatzes für schlichte konforme Abblildungen*, Math. Zeit. **4** (1919), no. 3-4, 295-305.

[2] C. Gao, S. Zhou, *On a class of analytic functions related to the starlike functions*, Kyungpook Math. J. **45** (2005), 123-130.

[3] T. Hayami, S. Owa, *Generalized Hankel Determinant for Certain Classes*, Int. Journal of Math. Analysis **4** (2010), no. 52, 2573-2585.

[4] F. R. Keogh, E. P. Merkes, *A coefficient inequality for certain class of analytic functions*, Proc. Amer. Math. Soc. **20** (1969), no. 1, 8-12.

[5] W. Koepf, *On the Fekete-Szegö problem for close-to-convex functions*, Proc. Amer. Math. Soc. **101** (1993), no. 1, 89-95.

[6] J. Kowalczyk, E. Leś-Bomba, *On a subclass of close-to-convex functions*, Appl. Math. Letters **23** (2010), 1147-1151.

[7] R. J. Libera, E. J. Złotkiewicz, *Early coefficients of the inverse of a regular convex function*, Proc. Amer. Math. Soc. **85** (1982), no. 2, 225-230.

[8] A. E. Livingston, *The coefficients of multivalent close-to-convex functions*, Proc. Amer. Math. Soc. **21** (1969), no. 3, 545-552.

[9] C. Pommerenke, *On the coefficients and Hankel determinants of univalent functions*, Proc. Lond. Math. Soc. **41** (1966), no. 3, 111-122.

[10] M. S. Robertson, *Certain classes of starlike functions*, Michigan Math. J. **76** (1954), no. 1, 755-758.

Ewa KULIGOWSKA

Gdynia Maritime University (Gdynia)

Monte Carlo simulation of climate-weather change process at maritime ferry operating Baltic Sea area

The paper presents computer simulation technique applied to generating the varying in time climate-weather change process at the critical infrastructure operating area. The Monte Carlo method, under the assumption of semi-Markov model of this process, is used to obtain its realizations. The general algorithm and the procedure of this process characteristics evaluation are presented and applied to the climate-weather change process for a maritime ferry operating area at the Baltic Sea restricted waters of Gdynia bay. On the basis of the proposed algorithm, the computer calculations using $C\#$ programming language determining these characteristics are performed.

References

[1] F. Grabski, J. Jaźwiński, *Funkcje o losowych argumentach w zagadnieniach niezawodności, bezpieczeństwa i logistyki*, Wydawnictwa Komunikacji i Łączności, Warszawa, 2009.

[2] F. Grabski, *Semi-Markov Processes: Applications in System Reliability and Maintenance*, Elsevier, 2014.

[3] EU-CIRCLE Report D2.1-GMU3, *Modelling Climate-Weather Change Process Including Extreme Weather Hazards*, 2016.

[4] EU-CIRCLE Report D2.3-GMU2, *Identification Methods and Procedures of Climate-Weather Change Process Including Extreme Weather Hazards*, 2016.

[5] EU-CIRCLE Report D3.3-GMU3, *Critical infrastructure operating area climate-weather change process (C-WCP) including extreme weather hazards (EWH) C-WCP model*, 2016.

[6] K. Kołowrocki, *Reliability of Large Systems*, Elsevier, 2004.
[7] K. Kołowrocki, *Reliability of Large and Complex Systems*, Elsevier, 2014.
[8] K. Kołowrocki, E. Kuligowska, J. Soszyńska-Budny, *Monte Carlo simulation for optimization of object operation process and reliability*, Journal of KONBiN **4(24)** (2012), 79-92.
[9] K. Kołowrocki, E. Kuligowska, *Monte Carlo simulation application to reliability evaluation of port grain transportation system operating at variable conditions*, Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars **4** (2013), no. 1, 73-81.
[10] K. Kołowrocki, J. Soszyńska-Budny, *Reliability and Safety of Complex Technical Systems and Processes: Modeling-Identification-Prediction-Optimization*, Springer, 2011.
[11] D. P. Kroese, T. Taimre, Z. I. Botev, *Handbook of Monte Carlo Methods*, John Willey and Sons, Inc., Hoboken, New Jersey, 2011.
[12] N. Limnios, G. Oprisan, *Semi-Markov Processes and Reliability*, Birkhauser, Boston, 2005.
[13] E. Zio, M. Marseguerra, *Basics of the Monte Carlo Method with Application to System Reliability*, LiLoLe, 2010.

Ewa KULIGOWSKA

Gdynia Maritime University (Gdynia)

Monte Carlo simulation evaluation of complex system reliability

In the paper, the application of the Monte Carlo simulation method is proposed to the reliability evaluation of ageing multistate complex system related to its operation process, changing in time its reliability structure and its components reliability parameters. The general model linking the system operation process model with the system reliability model is constructed. The Monte Carlo simulation procedure of complex systems reliability characteristics determination in the form of an algorithm based on this general model is presented.

References

[1] F. Grabski, J. Jaźwiński, *Funkcje o losowych argumentach w zagadnieniach niezawodności, bezpieczeństwa i logistyki*, Wydawnictwa Komunikacji i Łączności, Warszawa, 2009.
[2] F. Grabski, *Semi-Markov Processes: Applications in System Reliability and Maintenance*, Elsevier, 2014.
[3] EU-CIRCLE Report D2.1-GMU3, *Modelling Climate-Weather Change Process Including Extreme Weather Hazards*, 2016.
[4] EU-CIRCLE Report D2.3-GMU2, *Identification Methods and Procedures of Climate-Weather Change Process Including Extreme Weather Hazards*, 2016.
[5] EU-CIRCLE Report D3.3-GMU1, *Modelling inside dependences influence on safety of multistate ageing systems - Modelling safety of multistate ageing systems*, 2016.
[6] EU-CIRCLE Report D3.3-GMU3, *Critical infrastructure operating area climate-weather change process (C-WCP) including extreme weather hazards (EWH) C-WCP model*, 2016.
[7] M. S. Habibullah, E. Lumanpauw, K. Kołowrocki, et al., *A computational tool for general model of operation processes in industrial systems operation processes*, Electronic Journal Reliability and Risk Analysis: Theory and Applications **2** (2009), no. 4, 181-191.
[8] K. Kołowrocki, *Reliability of Large Systems*, Elsevier, 2004.
[9] K. Kołowrocki, *Reliability of Large and Complex Systems*, Elsevier, 2014.
[10] K. Kołowrocki, E. Kuligowska, J. Soszyńska-Budny, *Monte Carlo simulation for optimization of object operation process and reliability*, Journal of KONBiN **4(24)** (2012), 79-92.

[11] K. Kołowrocki, E. Kuligowska, *Monte Carlo simulation application to reliability evaluation of port grain transportation system operating at variable conditions*, Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars **4** (2013), no. 1, 73-81.
[12] K. Kołowrocki, J. Soszyńska-Budny, *Reliability and Safety of Complex Technical Systems and Processes: Modeling-Identification-Prediction-Optimization*, Springer, 2011.
[13] E. Kuligowska, *Reliability analysis of a system subjected to two-state operation process*, Scientific Journals Maritime University of Szczecin **36** (2013), 108, 100-104.
[14] D. P. Kroese, T. Taimre, Z. I. Botev, *Handbook of Monte Carlo Methods*, John Willey and Sons, Inc., Hoboken, New Jersey, 2011.
[15] N. Limnios, G. Oprisan, *Semi-Markov Processes and Reliability*, Birkhauser, Boston, 2005.
[16] A. Renyi, *Probability Theory*, North-Holland Publishing Company, Amsterdam, 1970.
[17] E. Zio, M. Marseguerra, *Basics of the Monte Carlo Method with Application to System Reliability*, LiLoLe, 2010.

Adam LECKO, Barbara ŚMIAROWSKA

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Classes of univalent functions related to the Blaschke products

Some classes of univalent functions related to the Blaschke products are presented [1]. Growth and Distortion Theorems are shown. Radii of starlikeness and convexity generalizing well known results of [2] and [3] are computed.

References

[1] A. Lecko, B. Śmiarowska, *Classes of analytic functions related to the Blasche products, I*, (in preparation).
[2] T. H. MacGregor, *The radius of convexity for starlike functions of order* 1/2, Proc. Amer. Math. Soc. **14** (1963), no. 1, 71-76.
[3] T. H. MacGregor, *The radius of univalence of certain analytic functions*, Proc. Amer. Math. Soc. **14** (1963), no. 3, 514-520.

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Dynamical modelling for special ternary and quaternary nanostructures in relation to quickly moving metallic surfaces

Taking into account complex-analytical aspects of supersonic gas [cf. [1]], dynamical modelling for special ternary, and quaternary nanostructures in relation to quickly moving metallic surfaces is investigated [1–5].

References

[1] L. Bers, *Mathematical Aspects of Subsonic and Transonic Gas Dynamics*, Wiley, New York, 1958.

[2] J. Ławrynowicz, S. Marchiafava, F. L. Castillo Alvarado, A. Niemczynowicz, *(Para)quaternionic geometry, harmonic forms, and stochastical relaxation*, Publ. Math. Debrecen **84** (2014), no. 1-2, 205-220.

[3] J. Ławrynowicz, A. Niemczynowicz, M. Nowak-Kępczyk, L. M. Tovar Sánchez, *On an extension of harmonicity and holomorphy*, Cont. Math., Contemporary Mathematics (2015), 8 pp., 243-250.

[4] J. Ławrynowicz, M. Nowak-Kępczyk, A. Valianti, M. Zubert, *Physics of complex alloys – one dimensional relaxation problem*, Bull. Soc. Sci. Lettres Łódź Sér. Rech. Déform. **65** (2015), no. 2, 27-46.

[5] M. Nowak-Kępczyk, *An algebra governing reduction of quaternary structures to ternary structures* I-III, Bull. Soc. Sci. Lettres Łódź Sér. Rech. Déform. **64** (2014), no. 2, 3; **65** (2015), no. 3.

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Logiki niemonotoniczne

Logiki klasyczne są monotoniczne: zbiór konsekwencji rośnie monotonicznie wraz ze wzrostem zbioru przesłanek. Mówiąc inaczej, w logice monotonicznej dodanie nowej przesłanki nie może unieważnić dotychczas wyprowadzalnej konkluzji.

Własność monotoniczności nie zachodzi na ogół we wnioskowaniu zdroworozsądkowym, czyli wnioskowaniu, którym posługują się ludzie w życiu codziennym. Jeśli chcę rozpocząć podróż samochodem, muszę ustalić jego miejsce postoju. Jeśli nie mam dodatkowych danych, rozsądnie jest założyć, że samochód jest tam, gdzie go zaparkowałem. Ta konkluzja może okazać się błędna. Jednak jeśli jej nie przyjmę, nigdy nie rozpocznę podróży.

Rozwój logik niemonotonicznych nastąpił na początku lat osiemdziesiątych poprzedniego stulecia i przez kilkanaście lat problematyka ta była jednym z kluczowych tematów teoretycznej sztucznej inteligencji.

Na wykładzie przedstawię szeroko podstawy wnioskowania zdroworozsądkowego, krótko omówię podstawowe logiki niemonotoniczne oraz pokażę kilka pułapek związanych z tymi logikami.

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Extremal functions in weighted Bergman spaces and Beurling-type theorem

Let $\mathbf{D}$ denote the unit disk in the complex plane. For $-1 < \alpha < \infty$, the weighted Bergman space A^2_α is the space of functions f that are holomorphic in $\mathbf{D}$ and such that

$$\|f\|^2_\alpha = \int_{\mathbf{D}} |f(z)|^2 dA_\alpha(z) < \infty,$$

where $dA_\alpha(z) = (\alpha+1)\,(1-|z|^2)^\alpha\,\frac{dxdy}{\pi} = (\alpha+1)\,(1-|z|^2)^\alpha\,dA(z)$.

A sequence of points $\{a_n\}$ in the unit disk is called an A^2_α zero-set if there is a function in A^2_α which vanishes precisely on the set $\{a_n\}$. A closed subspace I of A^2_α is called invariant if $zf \in I$ whenever $f \in I$. Clearly, the subspace of A^2_α consisting of functions vanishing on a given A^2_α zero-set is an invariant subspace, the so-called zero based invariant subspace. H. Hedelman [4], P. Duren, D. Khavison, and others [2], [3] considered the following extremal problem

$$\sup\{\operatorname{Re} f(0) \,:\, f \in I,\ \|f\|_\alpha \leqslant 1\}. \tag{1}$$

It is known that for $-1 < \alpha < \infty$ the extremal problem has a unique solution [5].

For any subset $E \subset A^2_\alpha$, let $[E]$ be the smallest invariant subspace containing E. $[E]$ is called the invariant subspace generated by E. In 1996 Aleman, Richter and Sundberg [1] proved the so called Beurling-type theorem for A^2_0 which states that if I is an invariant subspace of A^2_0, then $I = [I \ominus zI]$. This theorem has been extended by Shimorin in [7] and [8] for A^2_α with $\alpha \in (-1, 1]$. It follows from Hedenmalm and Zhu [6] that such a theorem fails for $\alpha > 4$.

We derive an explicit formula for the extremal function (i.e., the solution of extremal problem (1)) for the invariant subspace of A^2_α consisting of functions with a single zero of multiplicity at least k, $k = 1, 2, \ldots$ This extremal function is expressed in terms of a hypergeometric function. We also show that the extremal function for the invariant subspace of A^2_4 consisting of functions with one at least double zero can have two additional zeros in the unit disk. This implies the failure of the Beurling-type theorem for $\alpha = 4$.

The talk is based on joint work with R. Rososzczuk and M. Wołoszkiewicz-Cyll

References

[1] A. Aleman, S. Richter, C. Sundberg, *Beurling's Theorem for the Bergman space*, Acta Math. **177** (1996), 275-310.

[2] P. Duren, D. Khavinson, H. S. Shapiro, C. Sundberg, *Contractive zero-divisors in Bergman spaces*, Pacific J. Math. **157** (1993), 37-56.

[3] P. Duren, A. Schuster, *Bergman spaces*, American Mathematical Society, Providence, RI, 2004.

[4] H. Hedenmalm, *A factorization theorem for square area-integrable analytic functions*, J. Reine Angew. Math. **422** (1991), 45-68.

[5] H. Hedenmalm, B. Korenblum, K. Zhu, *Theory of Bergman spaces*, Springer-Verlag, New York, 2000.

[6] H. Hedenmalm, K. Zhu, *On the failure of optimal factorization for certain weigted Bergman spaces*, Complex Variables Theory Appl. **19** (1992), 165-176.

[7] S. Shimorin, *Wold-type decompositions and wandering subspaces for operators close to isometries*, J. Reine Angew. Math. **531** (2001), 147-189.

[8] S. Shimorin, *On Beurling-type theorems in weighted l^2 and Bergman spaces*, Proc. Amer. Math. Soc. **131** (2003), no. 6, 1777-1787.

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Analiza danych związanych z gospodarką odpadami

W dobie nagromadzenia ogromnych ilości danych, różnodziedzinowych baz danych, efektywna ich analiza i wyszukiwanie regularności stały się zadaniem niezmiernie istotnym. Eksploracja danych obarczona jest wieloma aspektami które ją utrudniają takimi jak bardzo duża liczba obserwacji, zbyt duża liczba atrybutów, nieistotność części zmiennych dla klasyfikacji obiektów, wzajemne współzależności zmiennych warunkowych, równoczesna obecność zmiennych różnego typu, występowanie niezdefiniowanych wartości zmiennych, występowanie błędnych wartości zmiennych, nierównomierny rozkład kategorii dla zmiennej celu [1, 2]. Prezentowane badania związane są ze wstępną analizą numeryczną danych opisujących zagadnienie gospodarki odpadami w kilkuset gminach z czterech województw polski wschodniej. W ramach wstępnej analizy przeprowadzono eksperymenty dotyczące selekcji najbardziej istotnych parametrów mających wpływ na gospodarowanie odpadami. W zależności od wskazanych parametrów można określić rodzaj administracyjny lub funkcjonalny badanej jednostki administracyjnej. Uzyskane wyniki stanowią również element szerszych badań związanych z budową ontologii dla tego zagadnienia [3, 4].

Selekcja cech stanowi bardzo znaną dziedzinę analizy danych. W literaturze znanych jest szereg dostępnych metod i algorytmów realizujących to zadanie. Proces selekcji ma na celu ograniczenie przestrzeni cech opisujących problem do zbioru cech mających największy wpływ na cechę decyzyjną (Most Relevant Feature Selection) lub zbiór wszystkich atrybutów które mają wpływ większy niż losowe wartości cech (All Relevant Feature Selection)[5, 6].

W procesie badawczym zastosowano wybrane algorytmy selekcji istotnych cech i miary oceny ich istotności. W głównej mierze wykorzystano algorytm Boruta bazujący na algorytmach lasu losowego. Ponadto zastosowano algorytmy bazujące na estymacji atrybutów z użyciem modeli drzew oraz reguł decyzji. Spośród pierwotnego zestawu atrybutów opisujących gminę został wyekstrahowany ograniczony ich zestaw.

References

[1] R. Kohavi, G. H. John, *Wrappers for feature subset selection*, Artif. Intell. **97** (1997), 273–324.

[2] M. L. Bermingham, R. Pong-Wong, A. Spiliopoulou, C. Hayward, I. Rudan, H. Campbell, A. F. Wright, J. F. Wilson, F. Agakov, P. Navarro, C. S. Haley, *Application of high-dimensional feature selection: evaluation for genomic prediction in man*, Sci. Rep. 5, (2015), Article number: 10312.

[3] P. Grochowalski, K. Pancerz, T. Szul, *Rough Set Based Approximations of Classes in the OWL Ontology of Places in Poland: Extended Abstract*, Proc. of the Workshop on Concurrency, Specification and Programming (CS&P'2016), Rostock, Germany, September 28-30, 2016.

[4] K. Pancerz, P. Grochowalski, A. Derkacz, *Towards the Ontology of Places in Poland: an Example of the Mazowieckie Voivodship*, Barometr Regionalny. Analizy i Prognozy **14**(3) (2016).

[5] W. R. Rudnicki, M. Wrzesień, W. Paja, *All Relevant Feature Selection Methods and Applications*, In: U. Stańczyk and C. J. Lakhmi (eds.) Feature Selection for Data and Pattern Recognition, pp. 11–28, Springer-Verlag Berlin Heidelberg, Berlin, 2015.

[6] W. Paja, M. Wrzesień, R. Niemiec, W. R. Rudnicki, *Application of all-relevant feature selection for the failure analysis of parameter-induced simulation crashes in climate models*, Geoscientific Model Development **9** (2016), 1065–1072.

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The Ontology of Places in Poland in the Description Logic Formalism

In [3], the ontology of places in Poland covering variety of aspects of places, mainly administrative and socio-economic, was introduced. The ontology is being implemented using the OWL 2 Web Ontology Language [4]. In the paper, we describe the representation of this ontology in a description logic formalism. A part of the ontology, covering the Lubelskie Voivodship, is considered. Description logics (DLs) [1] are a family of knowledge representation languages that are widely used in ontological modelling. Both parts of the created ontology, expressed in the description logic formalism, are described, the TBox containing intensional (terminological) knowledge and the ABox containing extensional (assertional) knowledge. Moreover, we show how to use rough sets to approximate secondary concepts by means of primary concepts defined in the description logic formalism (cf. [2]).

References

[1] F. Baader et al. (eds.), *The Description Logic Handbook. Theory, Implementation and Applications*, Cambridge University Press, 2010.

[2] P. Grochowalski, K. Pancerz, T. Szul, *Rough Set Based Approximations of Classes in the OWL Ontology of Places in Poland: Extended Abstract*, Proc. of the Workshop on Concurrency, Specification and Programming (CS&P'2016), Rostock, Germany, September 28-30, 2016.

[3] K. Pancerz, P. Grochowalski, A. Derkacz, *Towards the Ontology of Places in Poland: an Example of the Mazowieckie Voivodship*, Barometr Regionalny. Analizy i Prognozy **14**(3) (2016).

[4] B. Motik, P. F. Patel-Schneider, B. Parsia (eds.), *OWL 2 Web Ontology Language: Structural Specification and Functional-Style Syntax (Second Edition)*, W3C Recommendation, 2012.

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Alpha convex functions associated with conic domains

We define a new class $k-UM_{\alpha}[A,B]$ of Janowski type $k-$uniformly alpha convex functions. We use the method of differential subordinations theory to obtain some new results like sufficient condition, inclusion results, coefficient estimate and covering properties. The results presented here include a number of known results as their special cases.

References

[1] W. Janowski, *Some external problem for certain families of analytic functions, I.* Ann. Polon. Math. **28** (1973), 298-326.

[2] S. Kanas, A. Wiśniowska, *Conic regions and k-uniform convexity*, J. Comput. Appl. Math. **105** (1999), 327-336.

[3] S. Kanas, A. Wiśniowska, *Conic domains and starlike functions*, Rev. Roumaine Math. Pure. Appl. **45** (2000), 647-657.

[4] S. S. Miller, P. T. Mocanu, *Differential subordination and univalent functions*, Michigan. Math. J. **28**(1981), no. 2, 157-172.

[5] K. I. Noor, S. N. Malik, *On coefficient inequalities of functions associated with conic domains*, Comput. Math. Appl. **62** (2011), 2209-2217.

[6] M. Nunokawa, J. Sokół, *On order of strongly starlikeness in the class of uniformly convex functions*, Math. Nachr. **288** (2015), no. 8-9, 1003–1008.

[7] J. Stankiewicz, *Quelques problèmes extrémaux dans les classes des fonctions α-angulairement étoilées*, Ann. Univ. Mariae Curie-Skłodowska Sect. A **20** (1966), 59-75.

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Some Practical Applications of Fuzzy Set Theory

It is known that the fuzzy sets are the convenient tool for knowledge representation within information systems of different types [1, 2, 3]. Among variety of applications of fuzzy sets, it is possible to outline the analysis of the patient' symptoms in expert diagnostics systems. Determination of the diagnosis, based on fuzzy specifications of the symptoms, in such systems, requires using the mechanisms of fuzzy logical inference [4], which are built on ideas and methods of inductive mathematics. For this purpose, all knowledge, which can be usefull for determining of diagnosis, should be represented as fuzzy specification, i.e., ordered set of fuzzy instructions, where each instruction is a linguistic variable [5]. For diagnosis determination, system should analyze this specification and establish the approximate diagnosis.

For solving this problem, the algorithm of fuzzy logical inference, which calculates the approximate solution of described problem, is developed. Morover, the algorithm is implemented and integrated in the diagnostic system HOMEOPATH, which was described in [6, 7].

Fuzzy sets also can be used for recognizing of structure of proteins of different organization levels. However, as it was noted in [8], this problem is rather complicated. Different methods and approaches, including experimental (based on physics of chemical relations creation), machine learning (used the data bases of experimentally found secondary structures as learning samples), probabilistic (on the basis of the Bayes procedures and Markov chains) are used for solving this problem.

As the example of fuzzy sets application in bioinformatics, the method for recognition of the secondary structure of DNA, using fuzzy systems of logical inference, is proposed. The main idea of the method is a building of the fuzzy system of logical inference, which is using random amino acid sequence would define (as an fuzzy set) the secondary structure of central remainder (of the amino acid) of the input sequence. For this purpose, at first it is necessary to design the fuzzy specification of the problem according to learning samples. The second stage is building of the fuzzy system of logical inference, which would generate the correct output data according to random input values. For this purpose, the algorithm of fuzzy logical inference, which puts in accordance to each set of learning data the fuzzy rule of the logical inference, is proposed.

The proposed approach to solving problems (based on fuzzy models) allows to simplify the methods of solving problems. But there is a necessity for additional studies of the results reliability. Bayes' theorem offers an approach to the assessment of the reliability of the results and has achieved some success in expert systems in the last 20 years, and we will briefly describe it here. The rule itself is based on conditional probabilities. We write the probability the B is true given that P is true as $p(B|P)$. Bayes' rule reverses this, and is

$$p(A_k|B) = p(B|A_k) \cdot \frac{p(A_k)}{p(B)}$$

or eqivalently, since $p(B) = \sum p(B|A_i) \cdot p(A_i)$,

$$p(A_k|B) = \frac{p(B|A_k) \cdot p(A_k)}{\sum p(B|A_i) \cdot p(A_i)}.$$

Given a probability [9] distribution in the space X Bayes' recognition procedure allows to evaluate the reliability of the fuzzy inference system outputs by analogy with [10].

References

[1] D. Dubois, H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, Inc., 1980.

[2] G. J. Klir, B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Prentice Hall, 1995.

[3] W. Siler, J. J. Buckley, *Fuzzy Expert Systems and Fuzzy Reasoning*, John Wiley & Sons, Inc., 2005

[4] A. I. Provotar, A. V. Lapko, A. A. Provotar, *Fuzzy Inference Systems and Their Applications*, Cybernetics and Syst. Anal. **49** (2013), 517-525.

[5] L. A. Zadeh, *Fuzzy Sets as a Basis for a Theory of Possibility*, Fuzzy Sets and Syst. **1** (1978), 3-28.

[6] L. Katerynych, A. Provotar, *Neural Networks Diagnostics in Homeopath System*, Int. J. Inform. Theories & Appl. **15** (2008), 89-93.

[7] L. Katerynych, A. Provotar, *Homeopath: Diagnostic information system*, J. of Math. and Appl. **32** (2010), 75-84.
[8] A. Lesk, *Introduction to Bioinformatics*, 3rd ed., Oxford University Press, 2008.
[9] J. J. Buckley, *Fuzzy Probabilities: New Approach and Applications*. Physica-Verlag, 2003.
[10] A. M. Gupal, I. V. Sergienko, *Optimal Recognition Procedures* (in Russian), Naukova Dumka, 2008.

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Extremal functions in weighted Bergman spaces A^p_α for $-1 < \alpha < 0$ and $0 < p < \infty$

For a fixed $a \in \mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}$, $a \neq 0$, let $I^{p,\alpha}_{k\times a}$ be the invariant subspace in weighted Bergman space A^p_α consisting of functions having a zero at $z = a$ of order at least k. We find an explicit formula for the solution of the following extremal problem

$$\sup\Big\{\operatorname{Re} f(0) : f \in I^{p,\alpha}_{k\times a},\ \|f\|_{A^p_\alpha} \leqslant 1\Big\}$$

in case when $0 < p < \infty$ and $-1 < \alpha < 0$.

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On some differential subordinations

The purpose of this work is to present a new geometric approach to some problems in differential subordination theory. We also discuss the new results closely related to the generalized Briot-Bouquet differential subordination.

Theorem. *Let $p(z) = 1 + \sum_{n=1}^{\infty} c_n z^n$, be analytic in $|z| < 1$. If there exists a real number $|x| > 1$, $\pm x \notin p(|z| < 1)$, such that*

$$p(z) + \frac{zp'(z)}{p(z)} - \log\left|\frac{x+1}{x-1}\right|\left\{\log\frac{x+p(z)}{x-p(z)}\right\}^{-1} \prec G(z) := \frac{2\sqrt{3}z}{1-z^2}, \quad |z| < 1,$$

then p is a Carathéodory function, i.e.,

$$\mathfrak{Re}\{p(z)\} > 0, \quad |z| < 1.$$

Mathematics Subject Classifications (2010): primary 30C45; secondary 30C80.
Key words and phrases: analytic functions; convex functions; convex functions of order alpha; univalent functions; differential subordination.

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Petri nets and PN-system. Tools for modeling DEDSs

Modern discrete event dynamic systems (DEDSs) are highly parallel and distributed. They need to be analyzed from qualitative and quantitative points of view. Qualitative analysis concerns the verification of the correctness of the modeled system whereas qualitative one the evaluation of its efficiency.

Similarly as in many engineering fields, the design of DEDSs can be carried out using models. Petri nets allow the construction of models amenable both for correctness and efficiency analysis. Moreover they can be implemented using many different techniques (hardware, microprogrammed, software). Because of the graphical nature of net models, they are mostly self-documented specifications, making easier the communication among designers and users. Net models can be used during the entire life cycle of DEDSs. Examples of DEDSs are communication networks, computer systems and manufacturing systems. Nevertheless, the practical use of Petri nets is strongly dependent upon the existence of adequate computer tools - helping the user to handle all the details of a large and complex description of the modeled system.

The purpose of this paper is to present in a fairly intuitive manner the basic modeling concepts, the main techniques for qualitative analysis, the application of Petri nets, and own new graphical Petri net computer tools (called PN-system) for construction of net models as well as modification and analysis.

References

[1] R. David, H. Alla, *Petri Nets and Grafcet. Tools for modelling discrete event systems*, Prentice-Hall, New York, 1992.

[2] F. DiCesare et al., *Practice of Petri Nets in Manufacturing*, Chapman & Hall, London, 1993.

[3] J. L. Peterson, *Petri net theory and the modeling of systems*, Prentice-Hall, Englewood Cliffs, 1981.

[4] W. Reisig, *Sieci Petriego*, WN-T, Warszawa, 1988.

[5] Z. Suraj, M. Szpyrka, *Sieci Petriego i PN-tools. Narzędzia do konstrukcji i analizy sieci Petriego*, Wyd. WSP, Rzeszów, 1999.

[6] Z. Suraj, B. Komarek, *GRAF. System graficznej konstrukcji i analizy sieci Petriego*, Akademicka Oficyna Wydawnicza PLJ, Warszawa, 1994.

[7] M. Zhou, F. DiCesare, *Petri Net Synthesis for Discrete Event Control of Manufacturing Systems*, Kluwer Academic Publishers, Boston, 1993.

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Fragmentation process in continuum

An individual-based model of an infinite system of point particles in $\mathbf{R}^d$ introduced in [1] is discussed. In this model, each particle randomly produces a finite number of new

particles ('cloud') and disappears afterwards. The phase space for this model is the set Γ of all locally finite subsets of $\mathbb{R}^d$. The system's states are probability measures on Γ. To characterise such states we use appropriate real functions F on Γ (called observables) the evolution of which is obtained by means of the Kolmogorov equation

$$\frac{d}{dt}F_t = LF_t, \qquad F_t|_{t=0} = F_0, \qquad t > 0.$$

The 'generator' L specifies the model. The Markov evolution of the states is described in terms of their correlation functions in a scale of Banach spaces. We prove the existence and uniqueness of solutions of the corresponding evolution equation by using the Ovcyannikov's method, cf. [2]. Employing the method used in [3] we also prove that the obtained solution corresponds to the unique state.

References

[1] A. Tanaś, *A continuum individual based model of fragmentation: dynamics of correlation functions*, Annales Universitatis Mariae Curie-Sklodowska, sectio A - Mathematica **69**(2) (2015), 73-83.
[2] D. Finkelshtein, Y. Kondratiev, Y. Kozitsky, 0. Kutoviy, *The statistical dynamics of a spatial logistic model and the related kinetic equation*, Mathematical Models and Methods in Applied Sciences **25** (2015), 343-370.
[3] Y. Kondratiev, Y. Kozitsky, *The Evolution of States in a Spatial Population Model*, Journal of Dynamics and Differential Equations (2016), 1-39, DOI 10.1007/s10884-016-9526-6.

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Modelling and identification of climate-weather change process at maritime ferry operating Baltic Sea area

The method of semi-Markov modelling of the climate-weather change process at a fixed area is presented. There are proposed procedures for estimating the unknown basic parameters of the climate-weather change process semi-Markov model like the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the climate-weather transitions between the climate-weather states and the distributions of the climate-weather change process conditional sojourn times at the climate-weather states.

The method of statistical data of the climate-weather change process realizations given in CSV file analysis and processing using R programing language is proposed and applied to identification unknown parameters of the climate-weather change process semi-Markov model of the maritime ferry operating at restricted Baltic Sea area.

References

[1] V. Barbu, N. Limnios, *Empirical estimation for discrete-time semi-Markov processes with applications in reliability*, Journal of Nonparametric Statistics **18** (2006), 7-8, 483-498.
[2] P. Biecek, *Przewodnik po pakiecie R*, 2014.
[3] EU-CIRCLE Report D2.1-GMU3, *Modelling Climate-Weather Change Process Including Extreme Weather Hazards*, 2016.

[4] EU-CIRCLE Report D2.3-GMU2, *Identification Methods and Procedures of Climate-Weather Change Process Including Extreme Weather Hazards*, 2016.
[5] K. Kołowrocki, *Reliability of Large Systems*, Elsevier, 2004.
[6] K. Kołowrocki, *Reliability of Large and Complex Systems*, Elsevier, 2014.
[7] K. Kołowrocki, J. Soszyńska-Budny, *Reliability and Safety of Complex Technical Systems and Processes: Modeling-Identification-Prediction-Optimization*, Springer, 2011.
[8] N. Limnios, G. Oprisan, *Semi-Markov Processes and Reliability*, Birkhauser, Boston, 2005.
[9] N. Limnios, B. Ouhbi, A. Sadek, *Empirical estimator of stationary distribution for semi-Markov processes*, Communications in Statistics-Theory and Methods **34** (2005), no. 4, 987-995.
[10] J. Soszyńska, K. Kołowrocki, A. Blokus-Roszkowska, S. Guze, *Identification of complex technical system components safety models*, Summer Safety and Reliability Seminars, June 20-26, 2010, Gdańsk-Sopot, Poland; Journal of Polish Safety and Reliability Association **1** (2010), no. 2, 399-496.

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Investigation of dependencies in climate-weather change process at maritime ferry operating Baltic Sea area

The method of semi-Markov dependency analysis of the climate-weather change processes at different points of a fixed area is presented. There are proposed procedures for independence testing the basic parameters of the climate-weather change process semi-Markov model like the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the climate-weather transitions between the climate-weather states and the distributions of the climate-weather change process conditional sojourn times at the climate-weather states.

The method of statistical data of the climate-weather change processes realizations given in CSV file analysis and processing using R programing language is proposed and applied to independence testing parameters of the climate-weather change process semi-Markov model of the maritime ferry operating at restricted Baltic Sea area and at Karlskrona Port area.

References

[1] V. Barbu, N. Limnios, *Empirical estimation for discrete-time semi-Markov processes with applications in reliability*, Journal of Nonparametric Statistics **18** (2006), 7-8, 483-498.
[2] P. Biecek, *Przewodnik po pakiecie R*, 2014.
[3] EU-CIRCLE Report D2.1-GMU3, *Modelling Climate-Weather Change Process Including Extreme Weather Hazards*, 2016.
[4] EU-CIRCLE Report D2.3-GMU2, *Identification Methods and Procedures of Climate-Weather Change Process Including Extreme Weather Hazards*, 2016.
[5] K. Kołowrocki, *Reliability of Large Systems*, Elsevier, 2004.
[6] K. Kołowrocki, *Reliability of Large and Complex Systems*, Elsevier, 2014.
[7] K. Kołowrocki, J. Soszyńska-Budny, *Reliability and Safety of Complex Technical Systems and Processes: Modeling-Identification-Prediction-Optimization*, Springer, 2011.
[8] N. Limnios, G. Oprisan, *Semi-Markov Processes and Reliability*, Birkhauser, Boston, 2005.

[9] N. Limnios, B. Ouhbi, A. Sadek, *Empirical estimator of stationary distribution for semi-Markov processes*, Communications in Statistics-Theory and Methods **34** (2005), no. 4, 987-995.
[10] J. Soszyńska, K. Kołowrocki, A. Blokus-Roszkowska, S. Guze, *Identification of complex technical system components safety models*, Summer Safety and Reliability Seminars, June 20-26, 2010, Gdańsk-Sopot, Poland; Journal of Polish Safety and Reliability Association **1** (2010), no. 2, 399-496.

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Domains of n-valence for some classes of analytic functions

The univalence problems in the class T of typically real functions were considered by Golusin in [1]. He proved that the domain of local and global univalence for typically real functions coincide. The problems of univalence for the class $\mathrm{T}^{(2)}$ of odd typically real functions were investigated by Koczan and Zaprawa in [3, 4]. They proved that there can be infinitely many domains of univalence for odd typically real functions and they determined some of these domains.

We study the problems of n-valence in some classes of analytic functions. We determine some examples of the domains of n-valence for functions which are analytic with classical normalization and locally univalent.

First we consider the function

$$f(z) = \frac{1}{\pi} \operatorname{tg} \frac{\pi z}{1+z^2}, \quad z \in \mathbf{U} = \{z \in \mathbf{C} : |z| < 1\},$$

which was introduced by Goodman in [2]. The function f is typically real and locally univalent. We determine the radius of n-valence of the function f. We show that the function f has infinitely many domains of n-valence and we determine some of them.

Next we formulate a few conjectures concerning locally univalent and typically real functions and concerning locally univalent and odd typically real functions.

Finally, we study similar problems in the classes generated by functions which are not typically real. Namely, we consider the functions $h(z) = z + z^2,\ z \in \mathbf{U}$, and $g(z) = z + z^2 + z^3/3,\ z \in \mathbf{U}$. We determine the domains of univalence of the functions h and g.

References

[1] G. Golusin, *On typically real functions*, Mat. Sb. **27(69)** (1950), 201-218.
[2] A. W. Goodman, *The domain covered by a typicaly real function*, Proc. Amer. Math. Soc. **64** (1977), 233-237.
[3] L. Koczan, P. Zaprawa, *Domains of univalence for typically real odd functions*, Complex Variables **48** (2003), no. 1, 1-17.
[4] L. Koczan, P. Zaprawa, *On domains of univalence for the class of typically real functions*, Folia Sci. Univ. Tech. Resoviensis, Math. **25 (190)** (2001), 81-89.

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Growth function for some uniformly amenable groups

Definition. *A discrete group G is uniformly amenable if there exists a growth function $q : \mathbf{R}_+ \times \mathbf{N} \to \mathbf{N}$ such that for every $t > 0$ and every finite subset K of G there exists a finite subset U of G with $|U| \leqslant q(t, |K|)$ and $|KU| \leqslant (1 + t)|U|$.*

The word "uniformly" concerns the fact that the function q depends on t and on the cardinality $|K|$ of the set K but not the set K itself.

We give a very simple, elementary proof that every abelian group is uniformly amenable and we describe a growth function valid for all abelian groups and much smaller then previously known growth functions.

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Some properties of k-uniformly starlike functions

Let S denote the class of all functions f that are analytic and univalent in the open unit disk U and normalized by $f(0) = f'(0) - 1 = 0$. Given $k \in [0, 1]$, by k-UST we denote the class of all k-uniformly starlike functions. A function $f \in S$ is said to be k-uniformly starlike in U if the image of every circular arc γ contained in U with center at ζ, where $|\zeta| \leqslant k$, is starlike with respect to $f(\zeta)$. The class k-UST coincides with the class of all starlike functions for $k = 0$, and for $k = 1$ we get the class UST of uniformly starlike functions introduced by Goodman [1]. Thus the notion of k-uniform starlikeness of function f is intermediate between being starlike and uniformly starlike.

We present some basic properties of functions in k-UST such as analytic characterizations in terms of two complex variables, special members of this class, k-UST radius in the class of convex functions ([3]). Moreover we define the class of k-uniformly starlike functions in terms of dual sets and using duality technique of [2] we find the exact order of starlikeness of the class k-UST for $k < 1$ ([4]).

References

[1] A. W. Goodman, *On uniformly starlike functions*, J. Math. Anal. Appl. **155** (1991), 364-370.

[2] St. Ruscheweyh, *Duality for Hadamard products with applications to extremal problems for functions regular in the unit disc*, Trans. Amer. Math. Soc. **210** (1975), 63-74.

[3] A. Wiśniowska-Wajnryb, *On classes of uniformly starlike functions*, Ann. Polon. Math. **108**(1) (2013), 11-19.

[4] A. Wiśniowska-Wajnryb, *The order of starlikeness of the class of uniformly starlike functions*, Math. Nachrichten (to appear).

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Zastosowanie technologii informacyjnych w edukacji matematycznej studentów

Nie ulega wątpliwości fakt, że matematyka dla studentów kierunków niematematycznych (inżynierów, ekonomistów, lekarzy itp.) pełni rolę usługową. Nauczyciele akademiccy, planując treści nauczania, muszą mieć na względzie ciasne ramy czasowe wynikające z planu studiów. W większości przypadków są one niezadowalające. Mając na uwadze oba powyższe aspekty, należy dołożyć szczególnych starań, aby nauczanie matematyki było adekwatne do kierunku studiów i efektywne jednocześnie. Komputerowe wspomaganie tego procesu jest obecnie nie tylko możliwością ale wręcz koniecznością. Kształcenie wsparte wykorzystaniem technologii informacyjnych jest coraz częściej realizowane już w środowisku szkolnym. Naturalnym więc staje się fakt kontynuowania tego sposobu kształcenia również w środowisku akademickim.

W niniejszym opracowaniu przedstawię kilka zastosowań dostępnych pakietów matematycznych w nauczaniu matematyki studentów kierunków niematematycznych na przykładzie wybranych zagadnień analizy matematycznej.

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Coefficient estimates of the inverse functions in some classes of univalent functions

For selected classes of univalent functions coefficient estimates of the inverse functions are computed.

References

[1] R. M. Ali, *Coefficients of the Inverse of Strongly Starlike Functions*, Bull. Malay. Math. Sci. Soc. **26** (2003), 63-71.

[2] W. Ma, D. Minda, *A unified treatment of some special classes of univalent functions*, Proceedings of the International Conference on Complex Analysis at the Nankai Institute of Mathematics, 1992, International Press, 1994, pp. 157-169.

[3] E. Netanyahu, *The Minimal Distance of the Image Boundary from the Origin and the Second Coefficient of a Univalent Function in* $|z| < 1$, Arch. Rational Mech. Anal. **32** (1969), 100-112.